E2.5 Signals & Linear Systems

Tutorial Sheet 8 – DFT and z-transform

(Lectures 14 - 15)

1.* For a signal *f(t)* that is time-limited to 10 ms and has an essential bandwidth of 10 kHz, determine N_0, the number of signal samples necessary to compute a power of 2 DFT with a frequency resolution f_0 of at least 50 Hz. Explain if any zero padding is necessary.

2.* Choose appropriate values for N_0 and T and compute the DFT of the signal $e^{-t}u(t)$. (Note that the choice of N_0 and T is not unique; it will depend on your assumptions. What is important here is the reasoning that you use to arrive at your answer.)

3.** For the functions *f(t)* and *g(t)* shown in Fig. Q3, write the appropriate sequences f_k and g_k necessary for the computation of the fast convolution of *f(t)* and *g(t)*. Use T=1/8. (Note that fast convolution is achieved by converting *f(t)* and *g(t)* to frequency domain via fast Fourier Transform.)

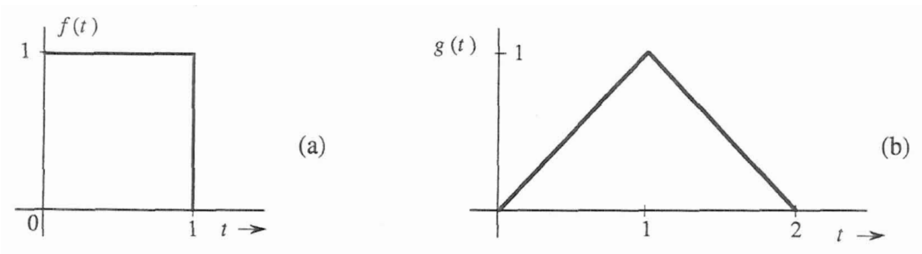

Figure Q3

4. Using the definition of z-transform, show that

(a)* $\gamma^{k-1}u[k-1] \Leftrightarrow \dfrac{1}{z-\gamma}$

(b)** $u[k-m] \Leftrightarrow \dfrac{z}{z^m(z-1)}$

(c)** $\dfrac{\gamma^k}{k!}u[k] \Leftrightarrow e^{\frac{\gamma}{z}}$

5. Using z-transform table given in the lecture notes, show that

(a)* $2^{k+1}u[k-1] + e^{k-1}u[k] \Leftrightarrow \dfrac{4}{z-2} + \dfrac{z}{e(z-e)}$

(b)** $k\gamma^k u[k-1] \Leftrightarrow \dfrac{\gamma z}{(z-\gamma)^2}$ (Hint: $u[k-1] = u[k] - \delta[k]$, $f[k]\delta[k] = f[0]\delta[k]$.)

(c)** $[2^{-k}\cos\dfrac{\pi}{3}k]\, u[k=1] \Leftrightarrow \dfrac{0.25(z-1)}{z^2 - 0.5z + 0.25}$

6. Find the inverse z-transform of

(a)* $X[z]=\dfrac{z(z-4)}{z^2-5z+6}$

(b)** $X[z]=\dfrac{z(e^{-2}-2)}{(e^{-2}-2)(z-2)}$

(c)** $X[z]=\dfrac{z(-5z+22)}{(z+1)(z-2)^2}$

(d)*** $X[z]=\dfrac{2z^2-0.3z+0.25}{z^2+0.6z+0.25}$

7.* Find the first three terms of *f[k]* using long division method given that

$$F[z]=\frac{2z^3+13z^2+z}{z^3+7z^2+2z+1}.$$

8.** For a discrete-time signal shown in Fig. Q8, show that

$$F[z]=\frac{1-z^{-m}}{1-z^{-1}}.$$

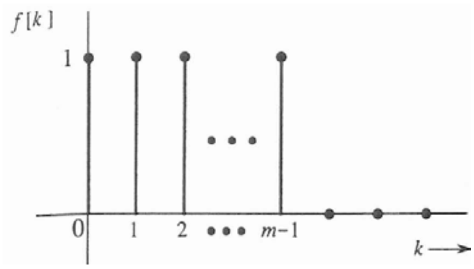

Figure Q8